Spring 2014	**Quiz 1**	**Date:**
K.Yaghi	**Math 201- Section X**	**Duration: 1 hour**

Problem 1 *(answer on page* 1 *of the booklet)*
Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence. (6 *pts each*)

a) $a_n = 2^n \frac{\sqrt[n]{n^3-1}}{n!}$ b) $b_n = \frac{n!\cos(7n^3+(-1)^n)}{n^n}$ c) $c_n = \frac{(-1)^n n^2+1}{n^2}$ d) $d_n = \frac{1+\frac{1}{2\ln 2}+\frac{1}{3\ln 3}+\cdots+\frac{1}{n\ln n}}{2\sqrt{n}+3\ln n}$

Problem 2 (*answer on pages* 2 & 3 *of the booklet*)
Which of the following series converge, and which diverge? When possible find the sum of the series. (7 *pts each*)

a) $\sum_{n=1}^{\infty} \frac{e^n}{\pi^{n+2}} + \frac{(-1)^n \pi^{2n}}{(2n)!}$ b) $\sum_{n=2}^{\infty} \frac{\sin(\frac{1}{n^{0.6}})}{n^{0.7}}$ c) $\sum_{n=1}^{\infty} (\cos[n\ ln\ (1+\frac{\pi}{2n})])^n$ d) $\sum_{n=2}^{\infty} \frac{(-1)^n n^3 \ln n}{2^n}$

Problem 3 (*answer on page* 4 *of the booklet*)
Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{(\sqrt{n+1})3^n}$$

For what values of x does the series converge absolutely? Conditionally? (20 *pts*)

Problem 4 (*answer on pages* 5 and 6 *of the booklet*)

a) (5 *pts*) Write a power series expansion for the function $f(x) = \ln(1+x^2)$ about the point $x = 0$. Also find the taylor polynomials $p2(x)$ and $p3(x)$ generated by $f(x)$ about the point $x = 0$.

b) (6 *pts*) Use the alternating series estimation theorem to estimate the error resulting from the approximation

$$\ln\left(\frac{5}{4}\right) \approx p3(?)$$

Does $p3(?)$ tend to be too small or too large?

c) (8 *pts*) Use taylor's theorem to prove that

$$\ln(1+x^2) \le x^2 \qquad \text{for} \quad 0 < x < 1$$

(*Hint: use taylor's theorem to prove that the error resulting from the approximation* $\ln(1+x^2) \approx x^2$ ***is strictly negative for*** $0 < x < 1$***)***

d) (5 *pts*) Decide if $\sum_{n=2}^{\infty} \ln(1+\frac{5^n (8000)^{3n}}{n!})$ converge or diverge? Justify your answer.

e) (4 *pts*) Find a power series expansion for

$$\int \frac{\ln(1+x^2)}{1+x^2}\, dx$$

(It's enough to find the first four terms)

Good Luck & Best Wishes
K. Yaghi